Calculus Warm Up #2-1

Review: A Series is the sum of a Sequence.

The Series represented by:

$$\sum_{n=1}^{4} 3n \quad = \quad 3 + 6 + 9 + 12$$

The sum is 30.

Write out the Series, then find the sum for:

1) $\displaystyle\sum_{n=1}^{5} (-1)^{n-1}\left(\frac{1}{n^2}\right)$

2) $\displaystyle\sum_{n=0}^{3} \sin^n\left(\frac{\pi}{4} + \pi n\right)$

Week 1 Classwork: Turned in on Friday

Warm Up on top

MC practice Part A, corrected with your chosen practice book problems attached.

FR practice #1 - 3

Week 1 HW Quiz: Tuesday, 4/3

p. 570, MC practice Part B, FR # 4 - 6

FR #4-6

4) a) $g(1)=0 \quad g'(1)=0$

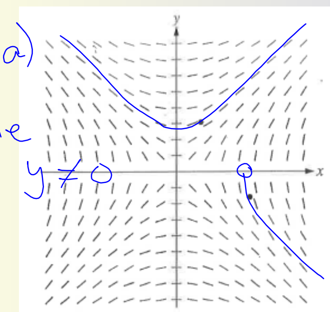
b) $1 < x < \pi$; where $g'(x)=f(x)$ is positive

c) Absolute Min @ $x=2\pi$

d) $g(1)=0$ and g has min @ $x=1$

$\therefore$ the graph of g is tangent to x-axis there.

5) a)



hole b/c $y \neq 0$

b) slope

$\frac{dy}{dx} = \frac{x}{y} = \frac{1}{2}$

tangent: $y-2 = \frac{1}{2}(x-1)$

c) $y = -\sqrt{x^2-8}$; $x > \sqrt{8}$

6a) $\lim_{x \to \infty} g(x) = 0$

b) where $g'\left(\frac{2}{3}\right)=0$ & $g''\left(\frac{2}{3}\right) = -$

$g'(x) = \frac{1-x-b}{e^x}$ $\qquad$ $g''(x) = \frac{-2+x+b}{e^x}$

$\boxed{b = \frac{1}{3}}$ $\qquad$ $g''\left(\frac{2}{3}\right) = -$ ✓

c) PI @ $x = 2-b$

$x > 0$, so $2-b > 0$

$\boxed{0 < b < 2}$

10.2 Infinite Series and Convergence

Geometric Series

Partial Sums

The nth Term Test for Divergence

Telescoping Series

Geometric Series

The sum of a Geometric Sequence

r = common ratio

$a_n = a_1 r^{n-1}$

n^{th} term

Series: $a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} + \ldots$

Converges if $|r| < 1$ $\quad S = \dfrac{a_1}{1 - r}$

$-1 < r < 1$

$$S = \sum_{n=1}^{\infty} ar^{n-1}$$

Find the infinite sum:

1) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$ $S = \frac{3}{1-\frac{1}{2}} = \boxed{6}$

$a_1 = 3 \quad r = \frac{1}{2}$

2) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots = \boxed{\frac{2}{3}}$

$a_1 = 1$

$r = -\frac{1}{2}$

$S = \frac{1}{1-\left(-\frac{1}{2}\right)}$

Writing a Geometric Series
for Repeating Decimals

Ex: $0.\overline{5} = 0.5 + 0.05 + 0.005 + \dots$

Express the nth term: $a_n = 0.5(0.1)^{n-1}$

$S = \sum_{n=1}^{\infty} 0.5(0.1)^{n-1}$

The series converges to $0.\overline{5}$

Write as a ratio of integers:

$S = \frac{0.5}{1-0.1}$

$= \frac{5}{9}$

let $x = 0.\overline{5}$

$10x = 5.\overline{5}$

$-\quad x \quad - .\overline{5}$

$9x = 5$

$x = \frac{5}{9}$

Partial Sums

The partial sums of a series form a sequence of real #'s. If the sequence has a limit as n →∞, then the infinite series converges.

Ex: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots$

Partial Sums

$S_1 = \frac{1}{2}$ $= 0.5$

$S_2 = \frac{1}{2} + \frac{1}{4}$ $= 0.75$

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ $= 0.875$

$S_4 =$ $= 0.9375$

$S_5 =$ $= 0.96875$

$= 0.984375$

$\lim_{n \to \infty} S_n = 1$

Series Converges

The nth Term Test for Divergence

Whenever an infinite series converges, the limit as n→∞ of the nth term = 0

* The difference between the terms getting added together is getting smaller and smaller (limit = 0) having little effect on the sum. The sum is converging on a real #, L.

This helps us to spot divergent series:

If $\lim_{n \to \infty} a_n \neq 0$, then the series diverges.

Telescoping Series

$$\sum_{n=1}^{\infty} \frac{2}{4n^2 - 1}$$

Rewrite a_n

$$\frac{2}{4n^2-1} = \frac{A}{2n+1} + \frac{B}{2n-1}$$

$$2 = A(2n-1) + B(2n+1)$$

for $n = \frac{1}{2} \rightarrow 2 = B(2) \longrightarrow B = 1$

$n = -\frac{1}{2} \rightarrow 2 = A(-2) \rightarrow A = -1$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

n^{th}

Write out the series:

$$S = \underbrace{\left(1 - \frac{1}{3}\right)}_{n=1} + \underbrace{\left(\frac{1}{3} - \frac{1}{5}\right)}_{n=2} + \underbrace{\left(\frac{1}{5} - \frac{1}{7}\right)}_{n=3} + \cdots \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$$

$= \boxed{1}$ converges

HW: p. 579,

1 - 43 odd

*Use partial fractions for # 23, 35, 37